

Title: Between the Extremes

Brief Overview:

Students will use the graphing calculator to analyze, write, and graph systems of inequalities from given data. Using real-life situations, students will collect and interpret data using graphing calculators.

Links to NCTM Standards:

- **Mathematics as Problem Solving**

The student will be able to demonstrate the ability to recognize and interpret information shown by the graph of a system of inequalities reflecting real-world situations.

- **Mathematics as Communication**

The student will be able to express either orally or in writing conclusions formulated from the graphs.

- **Mathematics as Reasoning**

The student will formulate logical arguments based on mathematical principles to justify responses to extension exercises and assessment questions.

- **Mathematical Connections**

The student will recognize the relationship between graphs of inequalities and topics in Biology and Physical Science.

- **Algebra**

The student will be able to represent situations that involve inequalities, and use graphs as tools to interpret those inequalities.

- **Functions**

The student will be able to represent and analyze relationships using graphs.

- **Statistics**

The student will be able to construct and draw inferences from graphs that show data from real-world situations.

- **Conceptual Underpinnings of Calculus**

The student will be able to determine the limiting processes by examining shaded portions between graphs.

Links to Maryland High School Mathematics Core Learning Goals:

- **1.1.1**

The student will recognize, describe, and extend patterns and functional relationships that are expressed numerically, algebraically, and geometrically.

- **1.1.2**

The student will represent patterns and functional relationships in a table, as a graph, and/or mathematical expression.

- **1.1.4**

The student will describe the graph of a non-linear function in terms of the basic concepts of maxima and minima, roots, limits, rate of change, and continuity.

- **1.2.1**
The student will determine the equation for a line, solve linear equations, and describe the solutions using numbers, symbols, and graphs.
- **1.2.2**
The student will solve linear inequalities and describe the solutions using numbers, symbols, and graphs.
- **1.2.3**
The student will solve and describe if and where two straight lines intersect using numbers, symbols, and graphs.
- **1.2.4**
The student will describe how the graphical model of a non-linear function represents a given problem and will estimate the solution.

Grade/Level:

Grades 9-12; Algebra I, Algebra II, Pre-Calculus, and Calculus

Duration/Length:

Two to three class periods (variable)

Prerequisite Knowledge:

Students should have working knowledge of the following skills :

- Transforming equations into slope intercept form
- Interpreting graphs of equations and inequalities
- Solving inequalities
- Using a TI-83 graphing calculator

Objectives:

Students will:

- graph inequalities on the TI-83 graphing calculator with appropriate shading.
- recognize and interpret the intersection of two inequalities in real-world situations.
- write inequalities and equations given real-world problems.

Materials/Resources/Printed Materials:

- Activity sheets
- TI-83 graphing calculators
- TI-83 overhead projector

Development/Procedures:

The drill reinforces putting equations in slope-intercept form. Activity 1 reviews solving inequalities and graphing on the TI-83 calculators. It introduces shading and intersection in teacher-student discussions. Activity 2 allows for practice with graphing and interpretation of system of inequalities.

Extension/Follow Up:

Extension #1 is a more advanced real-world problem dealing with weights, intake, and exercise. Extension #2 would be Activities 1 and 2 with equations in parabolic form.

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Activity #1

Drill/Warm Up

Put in slope intercept form and graph each of the following on the calculator.

- 1) $x + y = 6$
- 2) $3y = 6x + 9$
- 3) $2x - y = 8$
- 4) $3x - 2y = 5$

Teacher directed lesson

Part 1

From the drill turn off equations 2,3, and 4.

Press $Y=$.

Arrow over to $=$ and press ENTER until equal sign is no longer highlighted; do this for Y_2 , Y_3 , and Y_4 .

Then the equation is off and will not graph.

Part 2

What if we want drill question #1 to read $x + y > 6$? How does that change what appears on the $Y=$ screen? How will that change the graph? What do we need to do to get the graph we want? In what direction will the shading be?

This is how you get this:

Press $Y=$.

Left arrow to the front of Y_1 .

Press enter slowly until triangle in upper right appears.

Cursor over past $=$ sign.

Press graph.

Part 3

Return to $Y =$.

Turn off Y_1 using directions given above.

Go to Y_2 .

Arrow to $=$ sign.

Press enter so that the $=$ sign is highlighted.

This is now on.

Let #2 from the drill become $3y < 6x + 9$.

Ask questions similar to part 2.

How would you get this shading?

Have the students tell you how to achieve this.

Part 4

Turn Y_1 back on.

Press graph.

You now have the intersection of the two graphs.

Where is the intersection area?

What does it really represent?

Name a point in the intersection area?

How could you prove this point lies in the area of intersection analytically?

Activity #2

Group Activity

For each pair of inequalities, enter into the Y= screen and graph the area of intersection.

Group 1

$$\begin{aligned}2x + 4y &< 8 \\ 4x + 3y &> 7\end{aligned}$$

Group 2

$$\begin{aligned}3x + 9y &> 12 \\ 2x - 3y &< 4\end{aligned}$$

Group 3

$$\begin{aligned}x + 2y &< 6 \\ 5x - 2y &> 5\end{aligned}$$

Group 4

$$\begin{aligned}5x - 5y &< 4 \\ 3x - 4y &> 6\end{aligned}$$

Option to teacher:

Assign each group one problem or assign two problems per group or have each group do all the problems.

Activity # 3 Real-World Situation

Problem

Jeff is a member of the wrestling team in the 150-160 weight class. He is 154 lbs. so he must maintain his weight. He requires 2500 calories per day to maintain his bodily needs. He must exercise to burn off 1450 calories to maintain strength and stamina. For every 3500 calories you burn off you lose a pound. Let X for Y_1 be the calories over 2500 he consumes per day. Let X for Y_2 be the calories he works off over the 1450. Graph the equations representing calorie intake and exercise.

Discussion questions:

What equations goes into Y_1 and Y_2 ?

How does the 3500 calories play into each equation?

Why is the X in Y_1 added to 2500?

Why is the X in Y_2 added to 1450?

If $(X + 2500)/3500$ is calorie intake, what do we want to be the relationship between Y_1 and $(X + 2500)/3500$?

If $(X + 1450)/3500$ is calories usage, what do we want to be the relationship between Y_2 and $(x + 1450)/3500$?

Teacher Notes

Window to use

$X_{\min} = -2500$

$X_{\max} = 5000$

$X_{\text{scl}} = 100$

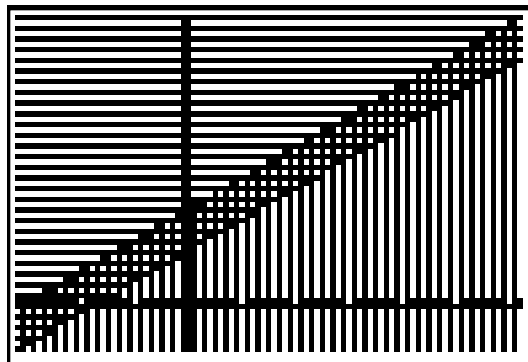
$Y_{\min} = -2$

$Y_{\max} = 2$

$Y_{\text{scl}} = 1$

The table can be utilized for differing calorie usage to see weight loss or gain.

The graph can be discussed about weight loss, gain, or maintenance.



Extension 1

Group work- practical application

Janice is a sophomore and feels that she is overweight. She needs 1500 calories to maintain her body weight provided she burns off 1000 calories in exercise. If she consumes 500 less calories per day and burns off 100 more in exercise per day, how long would it be before she loses 5 pounds? Remember 3500 calories equals one pound.

Questions for groups to write down and answer for either class discussion or collecting and grading.

- 1) What equation goes in Y_1 if X is the number of days she consumes less calories?
- 2) What equation goes in Y_2 if she exercises more?
- 3) What weight does she want to be?
- 4) Can she get there by diet alone?
- 5) Can she get there by exercise alone?

Activity #1

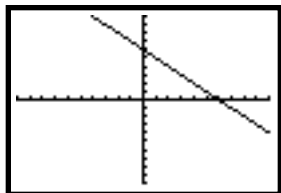
Answer Key

Drill/Warm Up

Put in slope intercept form and graph each of the following on the calculator.

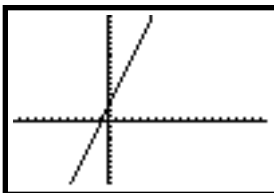
1) $x + y = 6$

ANSWER $Y = -X + 6$



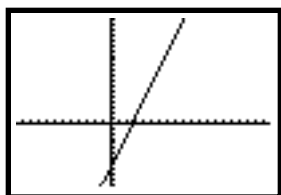
2) $3y = 6x + 9$

ANSWER $Y = 2X + 3$



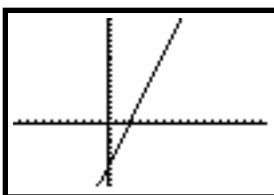
3) $2x - y = 8$

ANSWER $Y = 2X - 8$



4) $3x - 2y = 5$

ANSWER $Y = 1.5 X - 2.5$

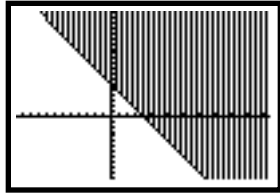


ACTIVITY #1 KEY CONTINUED

What if we want drill question #1 to read $x+y > 6$? How does that change what appears on the Y= screen? How will that change the graph? What do we need to do to get the graph we want? In what direction will the shading be?

The Y = screen stays the same.

Change the marking that appears in front of Y = to an upper right triangle.

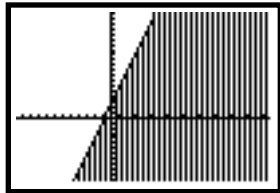


Let #2 from the drill become $3y < 6x + 9$.

How would you get this shading?

Have the students tell you how to achieve this.

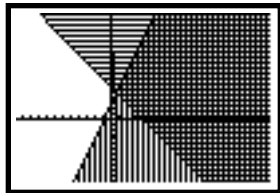
Change the marking that appears in front of Y = to an lower left triangle.



Turn Y₁ back on.

Press graph.

You now have the intersection of the two graphs.



Where is the intersection area?

What does it really represent?

Name a point in the intersection area?

How could you prove this point lies in the area of intersection analytically?

Answers will vary according to each student.

Activity #2

Answer Key

Group Activity

For each pair of inequalities, enter into the Y= screen and graph the area of intersection.

Group 1

$$\begin{aligned} 2x + 4y &< 8 \\ 4x + 3y &> 7 \end{aligned}$$

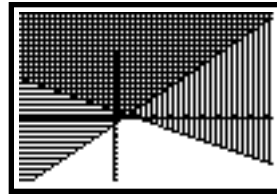
ANSWER
 $Y < (-1/2)X + 2$
 $Y > (-4/3)X + (7/3)$



Group 2

$$\begin{aligned} 3x + 9y &> 12 \\ 2x - 3y &< 4 \end{aligned}$$

ANSWER
 $Y > (-1/3)X + (4/3)$
 $Y < (2/3)X - (4/3)$



Group 3

$$\begin{aligned} x + 2y &< 6 \\ 5x - 2y &> 5 \end{aligned}$$

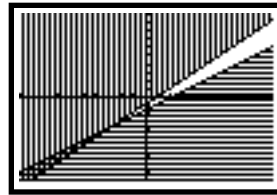
ANSWER
 $Y < (-1/2)X + 3$
 $Y < (5/2)X - (5/2)$



Group 4

$$\begin{aligned} 5x - 5y &< 4 \\ 3x - 4y &> 6 \end{aligned}$$

ANSWER
 $Y > X - (4/5)$
 $Y < (3/4)X - (3/2)$



Activity # 3 Real-World Situation

Problem

Jeff is a member of the wrestling team in the 150-160 weight class. He is 154 lbs. so he must maintain his weight. He requires 2500 calories per day to maintain his bodily needs. He must exercise to burn off 1450 calories to maintain strength and stamina. For every 3500 calories you burn off you lose a pound. Let X for Y_1 be the calories over 2500 he consumes per day. Let X for Y_2 be the calories he works off over the 1450. Graph the equations representing calorie intake and exercise.

Discussion questions:

What equations goes into Y_1 and Y_2 ?

How does the 3500 calories play into each equation?

Why is the X in Y_1 added to 2500?

Why is the X in Y_2 added to 1450?

If $(X + 2500)/3500$ is calorie intake, what do we want to be the relationship between Y_1 and $(X + 2500)/3500$?

If $(X + 1450)/3500$ is calories usage, what do we want to be the relationship between Y_2 and $(X + 1450)/3500$?

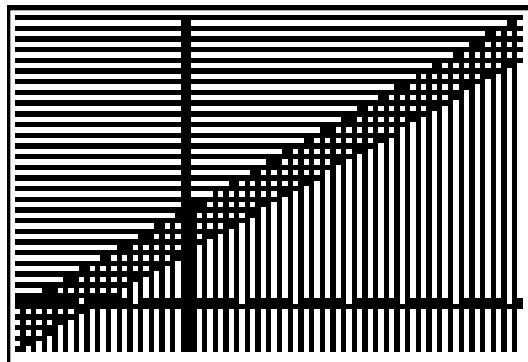
Selected answers

$Y_1 = (X + 2500)/3500$ X is the extra calories consumed.

$Y_2 = (X + 1450)/3500$ X is extra calories burned off.

$Y_1 < (X + 2500)/3500$

$Y_2 > (X + 1450)/3500$



Extension 1

Group work- practical application

Janice is a sophomore and feels that she is overweight. She needs 1500 calories to maintain her body weight provided she burns off 1000 calories in exercise. If she consumes 500 less calories per day and burns off 100 more in exercise per day, how long would it be before she loses 5 pounds? Remember 3500 calories equals one pound.

Questions for groups to write down and answer for either class discussion or collecting and grading.

- 1) What equation goes in Y_1 if X is the number of days she consumes less calories?
- 2) What equation goes in Y_2 if she exercises more?
- 3) What weight does she want to be?
- 4) Can she get there by diet alone?
- 5) Can she get there by exercise alone?

ANSWER KEY

From original problem 160 days ($158 < X < 190$)

- 1) $Y_1 = ((500 - 500) / 3500) X$
- 2) $Y_2 = ((400 - 500) / 3500) X$
- 3) 5 pounds lighter than she was.
- 4) No, diet alone maintains her present weight.
- 5) No, she must consume less and exercise to lose weight.

Performance Assessment

Teacher's Guide

Introduction

Students will use the graphing calculator to analyze, write, and graph systems of inequalities from given data. Using real-life situations, students will collect and interpret data using graphing calculators.

Objectives Covered

Students will:

- graph inequalities on the TI-83 graphing calculator with appropriate shading.
- recognize and interpret the intersection of two inequalities in real-world situations.
- write inequalities and equations given real-world problems.

Tools/Materials Needed for Assessment

Per student:

- Copy of assessment
- Calculator
- Writing instrument

Administering the Assessment

Distribute the necessary materials to each student.

SAY: *Today, you will use the graphing calculator with inequalities to deal with weight loss problems. Read your assessment sheet.*

Allow students time to read the assessment sheets.

SAY: *Are there any questions? You have the entire period to complete this task. You may begin.*

Performance Assessment

Student Response Sheet

- Mark weighs 165 pounds, does not like to exercise, but enjoys eating.
- His ideal weight should be 150 pounds.
- The caloric value of 3500 calories equals one pound.
- Mark needs to consume at least 1750 calories per day to maintain his metabolism.
- At present he is consuming about 2500 calories a day.
- Mark likes to take walks, he generally walks an hour a day.
- Walking an hour can burn 350 calories.

1) How long will it take Mark to reach his goal, if he cuts down to 1800 calories a day?

- a) 100 days b) 125 days c) 150 days d) 175 days

2) What do you enter into your Y_1 screen, if he reduces his calorie intake by 250?

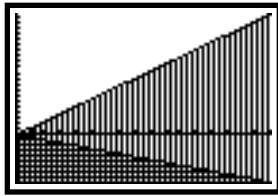
- a) $((165 - 150) / 3500) X$ b) $((250 - 165) / 3500) X$
c) $((750 - 500) / 3500) X$ d) $((2500 - 1750) / 3500) X$

3) What did you enter into your Y_2 screen, if he reduces his calorie intake by 500?

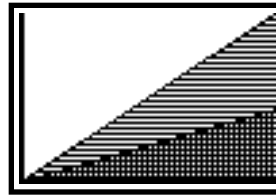
- a) $((500 - 165) / 3500) X$ b) $((750 - 850) / 3500) X$
c) $((1750 - 2500) / 3500) X$ d) $((2500 - 3500) / 3500) X$

4) Which graph represents the problem?

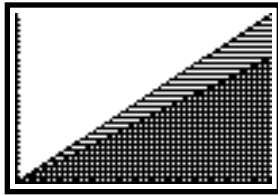
a)



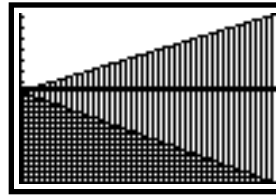
b)



c)



d)



5) Mark needs to lose 15 pounds in 90 days. Since walking still consumes 350 calories per hour and he walks one hour each day, how much must he cut his calorie intake by to reach his goal?

- a) 150 b) 400 c) 800 d) 1000

Energy Expended In Various Activity

<u>Activity</u>	<u>Expended Calories/hr</u>
Watching TV	25
Walking	350
Weight training	400
Bike riding	450
Basketball	550

6) If Mark can only reduce his calorie intake by 150, can he lose weight by playing basketball 1.5 hours per day? Explain your answer. Full credit can be achieved only by using the graphing calculator, sketching the graph, listing equations entered, and presenting written explanation and interpretation of information.

7) Mark increases his walking to 1.5 hours per day. Compare what happens to his weight if he reduces his food consumption by 300 calories instead 200 calories. Full credit can be achieved only by using the graphing calculator, sketching the graph, listing equations entered, and presenting written explanation and interpretation of information.

Performance Assessment

Scoring Guide

- Mark weighs 165 pounds, does not like to exercise, but enjoys eating.
- His ideal weight should be 150 pounds.
- The caloric value of 3500 calories equals one pound.
- Mark needs to consume at least 1750 calories per day to maintain his metabolism.
- At present he is consuming about 2500 calories a day.
- Mark likes to take walks, he generally walks an hour a day.
- Walking an hour can burn 350 calories.

1) How long will it take Mark to reach his goal, if he cuts down to 1800 calories a day?

- a) 100 days b) 125 days c) 150 days d) 175 days

ANSWER d) 175 days

2) What do you enter into your Y_1 screen, if he reduces his calorie intake by 250?

- a) $((165 - 150) / 3500) X$ b) $((250 - 165) / 3500) X$
c) $((750 - 500) / 3500) X$ d) $((2500 - 1750) / 3500) X$

ANSWER c) $((750 - 500) / 3500) X$

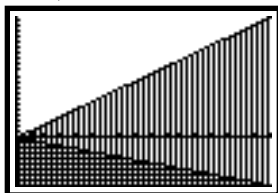
3) What did you enter into your Y_2 screen, if he reduces his calorie intake by 500?

- a) $((500 - 165) / 3500) X$ b) $((750 - 850) / 3500) X$
c) $((1750 - 2500) / 3500) X$ d) $((2500 - 3500) / 3500) X$

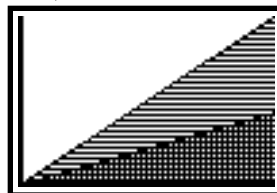
ANSWER b) $((750 - 850) / 3500) X$

4) Which graph represents the problem?

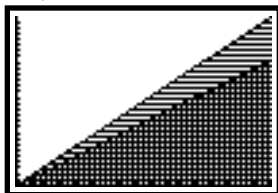
a)



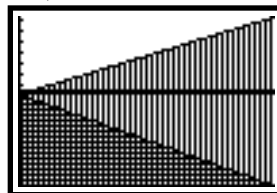
b)



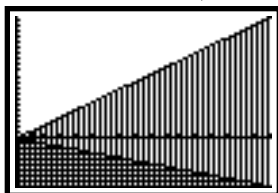
c)



d)



ANSWER a)



5) Mark needs to lose 15 pounds in 90 days. Since walking still consumes 350 calories per hour and he walks one hour each day, how much must he cut his calorie intake by to reach his goal?

- a) 150 b) 400 c) 800 d) 1000

ANSWER d) 1000

Point value for multiple choice question may be determined by the teacher.

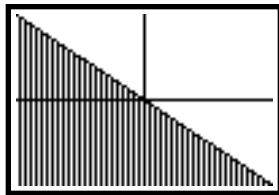
Energy Expended In Various Activity

Activity	Expended Calories/hr
Watching TV	25
Walking	350
Weight training	400
Bike riding	450
Basketball	550

6) If Mark can only reduce his calorie intake by 150, can he lose weight by playing basketball 1.5 hours per day? Explain your answer. Full credit can be achieved only by using the graphing calculator, sketching the graph, listing equations entered, and presenting written explanation and interpretation of information.

ANSWER: YES, HE WILL LOSE WEIGHT.

$$Y = ((750 - 975) / 3500) X$$



RUBRIC

4 POINTS-- STUDENT ANSWERS YES, GIVES CORRECT EXPLANATION, USES GRAPHING CALCULATOR, SHOWS GRAPH AND EQUATION USED.

3 POINTS-- STUDENT ANSWERS YES, GIVES CORRECT EXPLANATION USES ANALYTIC JUSTIFICATION WITH DATA. STUDENT DID NOT USE GRAPHING CALCULATOR.

2 POINTS-- STUDENT ANSWERS YES, WITH EXPLANATION INAPPROPRIATE FOR DATA GIVEN.

1 POINT-- STUDENT ANSWERS YES, NO OTHER EXPLANATION GIVEN.

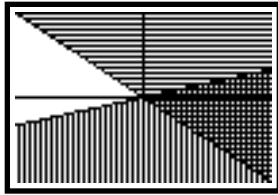
0 POINT -- ALL OTHER RESPONSES.

7) Mark increases his walking to 1.5 hours per day. Compare what happens to his weight if he reduces his food consumption by 300 calories instead 200 calories. Full credit can be achieved only by using the graphing calculator, sketching the graph, listing equations entered, and presenting written explanation and interpretation of information.

ANSWER: IF HE REDUCES CONSUMPTION BY 200 CALORIES HE WILL GAIN WEIGHT. IF HE REDUCES CONSUMPTION BY 300 CALORIES HE WILL LOSE WEIGHT.

$$Y_1 = ((750 - 725) / 3500) X$$

$$Y_2 = ((750 - 825) / 3500) X$$



RUBRIC

4 POINTS-- STUDENT MAKES ACCURATE COMPARISON BASED ON CORRECT INTERPRETATION OF GRAPH FROM CALCULATOR AND EQUATIONS ENTERED USING CORRECT DATA.

3 POINTS-- STUDENT MAKES ACCURATE COMPARISON BASED ON CORRECT INTERPRETATION OF ANALYTIC COMPUTATIONS USING CORRECT DATA. STUDENT DID NOT USE GRAPHING CALCULATOR

2 POINTS-- STUDENT MAKES ACCURATE COMPARISON BASED ON INTERPRETATION OF INCORRECT DATA.

1 POINT -- STUDENT MAKES ACCURATE COMPARISON WITH NO INTERPRETATION.

0 POINTS -- ALL OTHER RESPONSES.